OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4763**

Mechanics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 27 January 2010
Afternoon

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 **(a)** **(i)** Write down the dimensions of density, kinetic energy and power. **[3]**

A sphere of radius r is moved at constant velocity v through a fluid.

(ii) In a viscous fluid, the power required is $6\pi\eta r v^2$, where η is the viscosity of the fluid.

Find the dimensions of viscosity. **[3]**

(iii) In a non-viscous fluid, the power required is $k\rho^{\alpha} r^{\beta} v^{\gamma}$, where ρ is the density of the fluid and k is a dimensionless constant.

Use dimensional analysis to find α, β and γ. **[6]**

(b) A rock of mass 5.5 kg is connected to a fixed point O by a light elastic rope with natural length 1.2 m. The rock is released from rest in a position 2 m vertically below O, and it next comes to instantaneous rest when it is 1.5 m vertically above O.

Find the stiffness of the rope. **[6]**

2 **(a)** A uniform solid hemisphere of volume $\frac{2}{3}\pi a^3$ is formed by rotating the region bounded by the x-axis, the y-axis and the curve $y = \sqrt{a^2 - x^2}$ for $0 \leqslant x \leqslant a$, through 2π radians about the x-axis.

Show that the x-coordinate of the centre of mass of the hemisphere is $\frac{3}{8}a$. **[5]**

(b) A uniform lamina is bounded by the x-axis, the line $x = 1$, and the curve $y = 2 - \sqrt{x}$ for $1 \leqslant x \leqslant 4$. Its corners are A $(1, 1)$, B $(1, 0)$ and C $(4, 0)$.

(i) Find the coordinates of the centre of mass of the lamina. **[9]**

The lamina is suspended with AB vertical and BC horizontal by light vertical strings attached to A and C, as shown in Fig. 2. The weight of the lamina is W.

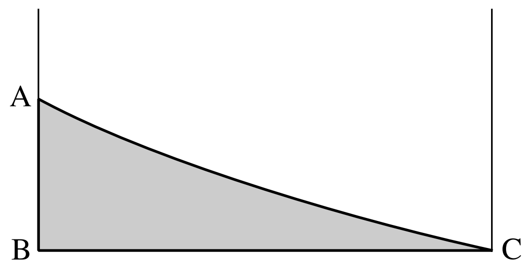

Fig. 2

(ii) Find the tensions in the two strings in terms of W. **[4]**

3 A particle P of mass 0.6 kg is connected to a fixed point O by a light inextensible string of length 1.25 m. When it is 1.25 m vertically below O, P is set in motion with horizontal velocity $6\,\text{m}\,\text{s}^{-1}$ and then moves in part of a vertical circle with centre O and radius 1.25 m. When OP makes an angle θ with the downward vertical, the speed of P is $v\,\text{m}\,\text{s}^{-1}$, as shown in Fig. 3.1.

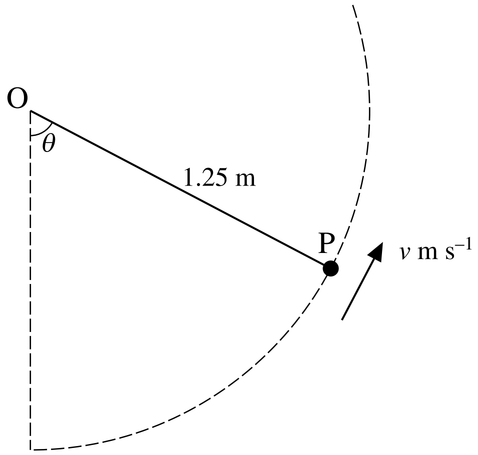

Fig. 3.1

(i) Show that $v^2 = 11.5 + 24.5\cos\theta$. **[3]**

(ii) Find the tension in the string in terms of θ. **[4]**

(iii) Find the speed of P at the instant when the string becomes slack. **[4]**

A second light inextensible string, of length 0.35 m, is attached to P, and the other end of this string is attached to a point C which is 1.2 m vertically below O. The particle P now moves in a horizontal circle with centre C and radius 0.35 m, as shown in Fig. 3.2. The speed of P is $1.4\,\text{m}\,\text{s}^{-1}$.

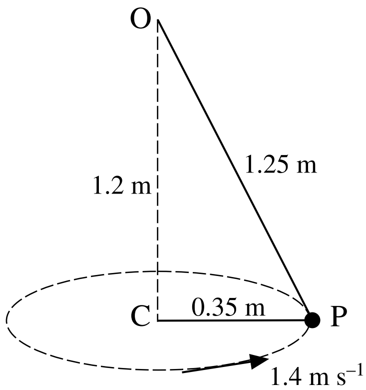

Fig. 3.2

(iv) Find the tension in the string OP and the tension in the string CP. **[7]**

[Question 4 is printed overleaf.]

4 Fig. 4 shows a smooth plane inclined at an angle of 30° to the horizontal. Two fixed points A and B on the plane are 4.55 m apart with B higher than A on a line of greatest slope. A particle P of mass 0.25 kg is in contact with the plane and is connected to A and to B by two light elastic strings. The string AP has natural length 1.5 m and modulus of elasticity 7.35 N; the string BP has natural length 2.5 m and modulus of elasticity 7.35 N. The particle P moves along part of the line AB, with both strings taut throughout the motion.

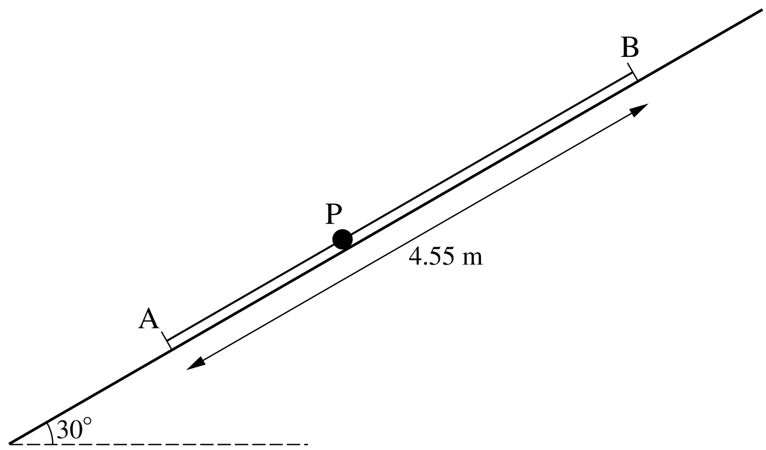

Fig. 4

(i) Show that, when AP = 1.55 m, the acceleration of P is zero. **[5]**

(ii) Taking AP = $(1.55 + x)$ m, write down the tension in the string AP, in terms of x, and show that the tension in the string BP is $(1.47 - 2.94x)$ N. **[3]**

(iii) Show that the motion of P is simple harmonic, and find its period. **[5]**

The particle P is released from rest with AP = 1.5 m.

(iv) Find the time after release when P is first moving *down* the plane with speed $0.2\,\mathrm{m\,s^{-1}}$. **[5]**

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1(a) (i)	[Density] $= \mathrm{M\,L^{-3}}$ [Kinetic Energy] $= \mathrm{M\,L^2\,T^{-2}}$ [Power] $= \mathrm{M\,L^2\,T^{-3}}$	B1 B1 B1	*(Deduct B1 for* $\mathrm{kg\,m^{-3}}$ *etc)*	3
(ii)	$\mathrm{M\,L^2\,T^{-3}} = [\eta]\mathrm{L}(\mathrm{L\,T^{-1}})^2$ $[\eta] = \mathrm{M\,L^{-1}\,T^{-1}}$	B1 M1 A1	For $[v] = \mathrm{L\,T^{-1}}$ *Can be earned in (iii)* Obtaining the dimensions of η	3
(iii)	$\mathrm{M\,L^2\,T^{-3}} = (\mathrm{M\,L^{-3}})^{\alpha}\,\mathrm{L}^{\beta}\,(\mathrm{L\,T^{-1}})^{\gamma}$ $\alpha = 1$ $-3 = -\gamma$ $\gamma = 3$ $2 = -3\alpha + \beta + \gamma$ $\beta = 2$	B1 cao M1 A1 M1 A1 A1	Considering powers of T *(No ft if* $\gamma = 0$ *)* Considering powers of L Correct equation *(ft requires 4 terms)* *(No ft if* $\beta = 0$ *)*	6
(b)	EE at start is $\frac{1}{2}k \times 0.8^2$ EE at end is $\frac{1}{2}k \times 0.3^2$ $\frac{1}{2}k \times 0.8^2 = \frac{1}{2}k \times 0.3^2 + 5.5 \times 9.8 \times 3.5$ Stiffness is $686\ \mathrm{N\,m^{-1}}$	M1 A1 A1 M1 F1 A1	Calculating elastic energy k may be $\frac{\lambda}{l}$ or $\frac{\lambda}{1.2}$ Equation involving EE and PE *(must have three terms)* (A0 for $\lambda = 823.2$)	6
				[18]

2 (a)	$\int \pi x y^2 \, dx = \int_0^a \pi x(a^2 - x^2)\, dx$	M1	*Limits not required*	
	$= \pi\left[\frac{1}{2}a^2x^2 - \frac{1}{4}x^4 \right]_0^a$	A1	For $\frac{1}{2}a^2x^2 - \frac{1}{4}x^4$	
	$= \frac{1}{4}\pi a^4$	A1		
	$\bar{x} = \dfrac{\frac{1}{4}\pi a^4}{\frac{2}{3}\pi a^3}$	M1		
	$= \frac{3}{8}a$	E1		5
(b) (i)	Area is $\int_1^4 (2-\sqrt{x})\, dx$	M1	*Limits not required*	
	$= \left[2x - \frac{2}{3}x^{3/2} \right]_1^4 \quad (= \frac{4}{3})$	A1	For $2x - \frac{2}{3}x^{\frac{3}{2}}$	
	$\int x y \, dx = \int_1^4 x(2-\sqrt{x})\, dx$	M1	*Limits not required*	
	$= \left[x^2 - \frac{2}{5}x^{\frac{5}{2}} \right]_1^4 \quad (= \frac{13}{5})$	A1	For $x^2 - \frac{2}{5}x^{\frac{5}{2}}$	
	$\bar{x} = \dfrac{13/5}{4/3} = \dfrac{39}{20} = 1.95$	A1		
	$\int \frac{1}{2}y^2 \, dx = \int_1^4 \frac{1}{2}(2-\sqrt{x})^2\, dx$	M1	$\int (2-\sqrt{x})^2\, dx$ *or* $\int \left((2-y)^2 - 1\right) y\, dy$	
	$= \left[2x - \frac{4}{3}x^{3/2} + \frac{1}{4}x^2 \right]_1^4 \quad (= \frac{5}{12})$	A2	For $2x - \frac{4}{3}x^{\frac{3}{2}} + \frac{1}{4}x^2$ *or* $\frac{3}{2}y^2 - \frac{4}{3}y^3 + \frac{1}{4}y^4$ Give A1 for two terms correct, or all correct with ½ omitted	
	$\bar{y} = \dfrac{5/12}{4/3} = \dfrac{5}{16} = 0.3125$	A1		9
(ii)	Taking moments about A $T_C \times 3 - W \times 0.95 = 0$	M1 A1	Moments equation (no force omitted) Any correct moments equation (May involve both T_A and T_C) Accept Wg or $W = \frac{4}{3}, \frac{4}{3}g$ here	
	$T_A + T_C = W$	M1	Resolving vertically (or a second moments equation)	
	$T_A = \dfrac{41}{60}W, \quad T_C = \dfrac{19}{60}W$	A1	*Accept* $0.68W, \quad 0.32W$	4
				[18]

3 (i)	By conservation of energy, $\frac{1}{2}\times0.6\times6^2-\frac{1}{2}\times0.6v^2=0.6\times9.8(1.25-1.25\cos\theta)$ $36-v^2=24.5-24.5\cos\theta$ $v^2=11.5+24.5\cos\theta$	M1 A1 E1	Equation involving KE and PE	3
(ii)	$T-0.6\times9.8\cos\theta=0.6\times\dfrac{v^2}{1.25}$ $T-5.88\cos\theta=0.48(11.5+24.5\cos\theta)$ $T=5.52+17.64\cos\theta$	M1 A1 M1 A1	For acceleration $\dfrac{v^2}{r}$ Substituting for v^2	4
(iii)	String becomes slack when $T=0$ $\cos\theta=-\dfrac{5.52}{17.64}$ ($\theta=108.2°$ or 1.889 rad) $v^2=11.5-24.5\times\dfrac{5.52}{17.64}$ Speed is 1.96 m s^{-1} (3 sf)	M1 A1 M1 A1 cao	 *May be implied* or $0.6\times9.8\times\dfrac{5.52}{17.64}=0.6\times\dfrac{v^2}{1.25}$ or $-0.6\times9.8\times\dfrac{v^2-11.5}{24.5}=0.6\times\dfrac{v^2}{1.25}$	4
(iv)	$T_1\cos\theta=mg$ $T_1\times\dfrac{1.2}{1.25}=0.6\times9.8$ (where θ is angle COP) Tension in OP is 6.125 N $T_1\sin\theta+T_2=\dfrac{mv^2}{0.35}$ $6.125\times\dfrac{0.35}{1.25}+T_2=\dfrac{0.6\times1.4^2}{0.35}$ Tension in CP is 1.645 N	M1 A1 A1 M1 F1B1 A1	Resolving vertically Horizontal equation (three terms) For LHS and RHS	7
				[18]

4(i)	$T_{AP} = \frac{7.35}{1.5} \times 0.05 \quad (= 0.245)$ $T_{BP} = \frac{7.35}{2.5} \times 0.5 \quad (= 1.47)$ Resultant force up the plane is $T_{BP} - T_{AP} - mg \sin 30°$ $= 1.47 - 0.245 - 0.25 \times 9.8 \sin 30°$ $= 1.47 - 0.245 - 1.225$ $= 0$ Hence there is no acceleration	M1 A1 A1 M1 E1	Using Hooke's law or $\frac{7.35}{1.5}(\text{AP} - 1.5)$ or $\frac{7.35}{2.5}(2.05 - \text{AP})$ Correctly shown	5
(ii)	$T_{AP} = \frac{7.35}{1.5}(0.05 + x) \quad (= 0.245 + 4.9x)$ $T_{BP} = \frac{7.35}{2.5}(4.55 - 1.55 - x - 2.5)$ $= 2.94(0.5 - x)$ $= 1.47 - 2.94x$	B1 M1 E1		3
(iii)	$T_{BP} - T_{AP} - mg \sin 30° = m\frac{d^2x}{dt^2}$ $(1.47 - 2.94x) - (0.245 + 4.9x) - 1.225 = 0.25\frac{d^2x}{dt^2}$ $\frac{d^2x}{dt^2} = -31.36x$ Hence the motion is simple harmonic Period is $\frac{2\pi}{\sqrt{31.36}} = \frac{2\pi}{5.6}$ Period is 1.12 s (3 sf)	M1 A2 E1 B1 cao	Equation of motion parallel to plane Give A1 for an equation which is correct apart from sign errors Must state conclusion. Working must be fully correct (cao) *If a is used for accn down plane, then $a = 31.36x$ can earn M1A2; but E1 requires comment about directions* *Accept* $\frac{5\pi}{14}$	5
(iv)	$x = -0.05 \cos 5.6t$ $v = 0.28 \sin 5.6t$ $-0.2 = 0.28 \sin 5.6t$ OR $0.2^2 = 31.36(0.05^2 - x^2)$ $x = (\pm)\ 0.035$ $0.035 = -0.05 \cos 5.6t$ M1 $5.6t = \pi + 0.7956$ Time is 0.703 s (3 sf)	M1 A1 M1 M1 A1cao	For $A \sin \omega t$ or $A \cos \omega t$ Allow $\pm 0.05 \sin/\cos 5.6t$ *Implied by* $v = \pm 0.28 \sin/\cos 5.6t$ Using $v = \pm 0.2$ to obtain an equation for t Fully correct strategy for finding the required time	5
				[18]

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General Comments

The standard of work on this paper was very high. Most candidates were able to demonstrate their competence in all the topics which were tested, and about three fifths of the candidates scored 60 marks or more (out of 72).

Comments on Individual Questions

1 Dimensional analysis and elastic energy

This question was answered well, with about one third of the candidates scoring full marks. The average mark was about 15 (out of 18).

(a) (i) Almost all candidates gave the dimensions of density and kinetic energy correctly, but about one third gave the wrong dimensions for power. Common misconceptions were that power may be calculated as (energy × time) or (force × distance).

(ii) The method for finding the dimensions of viscosity was well understood and usually applied accurately.

(iii) The method for finding the indices in the formula was very well known, although there were quite a few careless slips (usually sign errors) in this part.

(b) The great majority of candidates realised that they should apply conservation of energy, and used appropriate formulae for elastic and gravitational potential energy. Minor slips were sometimes made in the calculation of the terms, but by far the most common error was to neglect the elastic energy when the rock is at its highest point.

2 Centres of mass

This question was very well understood. About 40% of the candidates scored full marks, and the average mark was about 15.

(a) Almost all the candidates were able to derive the given result correctly.

(b) (i) Almost all candidates knew appropriate formulae for finding the centre of mass of a lamina, with just a very few confusing them with those for a solid of revolution. The integrations were usually done well, although common errors here were losing the factor ½ in the integral for the y-coordinate, and failing to multiply out $(2-\sqrt{x})^2$ correctly.

(ii) Most candidates knew that they should take moments to find the tensions in the strings. However, a very common error in this part was to proceed as if the x-coordinates of A and B were 0 instead of 1.

3 Circular motion

Circular motion has often been a cause of difficulty for many candidates in the past. However, this question was answered very well indeed, with nearly half the candidates scoring full marks.

(i) Most candidates obtained the given equation correctly. Just a few did not realise that conservation of energy was required, and tried to derive the result from the radial equation of motion.

(ii) Most candidates used the radial equation of motion to find the tension in the string correctly. The work was quite often spoilt by sign errors, or by omitting the component of the weight.

(iii) This was very well understood. Some candidates found the value of θ when the string becomes slack, but omitted to calculate the speed.

(iv) This problem, about a particle moving in a horizontal circle while attached to two strings, was answered much better than similar problems in the past. There were no common errors, and most candidates found both tensions correctly.

4 Simple harmonic motion

This was certainly found to be the most difficult question. Only about 15% of the candidates scored full marks, and the average mark was about 12.

(i) This was quite well done, with most candidates calculating the tensions in the given position and verifying that there is no resultant force parallel to the plane. Others considered a general position and formed an equation for the equilibrium position. Some tried to resolve horizontally or vertically, but all of these omitted the normal reaction from their calculations.

(ii) The tension in AP was quite often wrong, with the extension being taken as x or $(1.55 + x)$ instead of $(0.05 + x)$. However, the given expression for the tension in BP was almost always derived correctly.

(iii) Most candidates set up an equation of motion parallel to the plane, although there were very many sign errors here, and the component of the weight was sometimes omitted. A fair number of candidates did not state at what stage they had established that the motion is simple harmonic, but most knew how to find the period from the equation of motion.

(iv) Most candidates were able to obtain a time at which the speed is $0.2\,\mathrm{m\,s^{-1}}$, but finding the first time for which the particle has this speed when travelling down the plane proved to be quite challenging.